MATHEMATICS GRADE 12 MHF4U, ADVANCED FUNCTIONS **TEST Chapter 4** **TRIGONOMETRIC FUNCTIONS** Teacher: Mr Gugoiu	Date WINTER 2010 Name SOLUTIONS	$\frac{\quad}{50}$ =

The following questions are long answer questions. Show your work to get full marks.

1. Find the exact value of: [K/U 4 marks]

[2] a) $\sin\frac{19\pi}{6} = \sin\left(3\pi + \frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

[2] b) $\cos\frac{13\pi}{4} = \cos\left(3\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

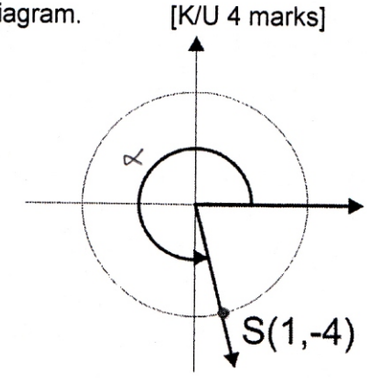
2. Find the value of all six trigonometric functions for the angle represented in the diagram. [K/U 4 marks]

$x = 1$
$y = -4$
$R = \sqrt{x^2+y^2}$
$= \sqrt{1+16}$
$R = \sqrt{17}$

$\sin\alpha = \frac{y}{R} = \frac{-4}{\sqrt{17}}$; $\csc\alpha = \frac{\sqrt{17}}{-4}$

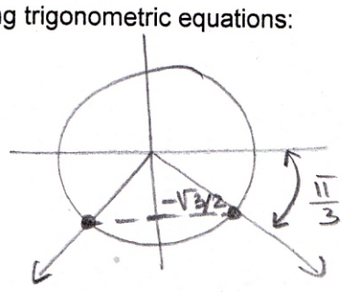
$\cos\alpha = \frac{x}{R} = \frac{1}{\sqrt{17}}$; $\sec\alpha = \sqrt{17}$

$\tan\alpha = \frac{y}{x} = -4$; $\cot\alpha = -\frac{1}{4}$



3. Solve the following trigonometric equations: [K/U 5 marks]

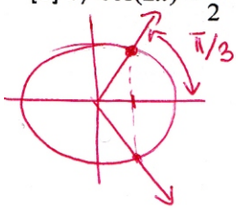
[2] a) $\sin x = -\frac{\sqrt{3}}{2}$



$x = -\frac{\pi}{3} + 2k\pi = \frac{5\pi}{3} + 2k\pi$
or
$x = -\frac{2\pi}{3} + 2k\pi = \frac{4\pi}{3} + 2k\pi$

$2\pi - \frac{2\pi}{3}$

[3] b) $\cos(2x) = \frac{1}{2}$



$2x = \pm\frac{\pi}{3} + 2k\pi \Rightarrow$

$x = \pm\frac{\pi}{6} + k\pi$

$x = \frac{5\pi}{6} + k\pi$
or
$x = \frac{\pi}{6} + k\pi$

3. Prove the following trigonometric identities: [A 5 marks]

[2] a) $\frac{1+\csc x}{\cot x} - \sec x = \tan x$

$LS = \frac{1+\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} - \frac{1}{\cos x} = \frac{\sin x + 1}{\cos x} - \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \tan x = RS$

[3] b) $\frac{\cos 2x}{\sin 2x + 1} = \frac{1-\tan x}{1+\tan x}$

$LS = \frac{\cos^2 x - \sin^2 x}{2\sin x\cos x + \sin^2 x + \cos^2 x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} = \frac{\cos x - \sin x}{\cos x + \sin x} =$

$= \frac{1-\tan x}{1+\tan x} = RS$

4. Find the exact value of $\cos\frac{7\pi}{12}$. [A 3 marks]

$\cos\frac{7\pi}{12} < 0 \quad \left(\frac{7\pi}{12} \in \text{II}\right)$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\cos\alpha = \pm\sqrt{\frac{1+\cos 2\alpha}{2}}$

$\cos\frac{7\pi}{12} = -\sqrt{\frac{1+\cos\frac{7\pi}{6}}{2}}$

$= -\sqrt{\frac{1+\cos(\pi+\pi/6)}{2}}$

$= -\sqrt{\frac{1-\cos(\pi/6)}{2}}$

$= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= -\frac{\sqrt{2-\sqrt{3}}}{2}$

$\therefore \cos\frac{7\pi}{12} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

$\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

$\therefore \cos\frac{7\pi}{12} = \frac{\sqrt{2}}{4}(1-\sqrt{3})$

5. Express $\sin(3x)$ in terms of $\sin x$. [A 3 marks]

$\sin(3x) = \sin(x+2x) = \sin x\cos 2x + \cos x \sin 2x$

$= \sin x(1-2\sin^2 x) + \cos x(2\sin x\cos x)$

$= \sin x - 2\sin^3 x + 2\sin x\cos^2 x$

$= \sin x - 2\sin^3 x + 2\sin x(1-\sin^2 x)$

$= \sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x$

$\therefore \sin(3x) = 3\sin x - 4\sin^3 x$

6. Simplify $\sin\left(x-\frac{\pi}{2}\right) - \cos(x+\pi) + \tan(\pi - x)$. Show your work. [A 3 marks]

$\sin\left(x-\frac{\pi}{2}\right) = \sin x\cos\frac{\pi}{2} - \cos x\sin\frac{\pi}{2} = -\cos x$

$\cos(x+\pi) = \cos x\cos\pi - \sin x\sin\pi = -\cos x$

$\tan(\pi - x) = \frac{\sin(\pi-x)}{\cos(\pi-x)} = \frac{\sin\pi\cos x - \cos\pi\sin x}{\cos\pi\cos x + \sin\pi\sin x} = \frac{\sin x}{-\cos x} = -\tan x$

$\therefore -\tan x$

7. Solve for α. [A 3 marks]

$\sin\alpha = 0.4 \quad ; \quad 0° \le \alpha \le 500°$

$\alpha_1 = \sin^{-1} 0.4 \simeq 23.578°$ ✓

$\alpha_2 = 180° - \alpha_1 \simeq 156.422°$ ✓

$\alpha_3 = \alpha_1 + 360° \simeq 383.578°$ ✓

$\alpha_4 = \alpha_2 + 360° \simeq 516.422°$ ×

$\therefore \alpha \in \{\alpha_1, \alpha_2, \alpha_3\}$

8. Solve the triangle ABC. [A 4 marks]

$20^2 = 15^2 + 12^2 - 2(15)(12)\cos B$

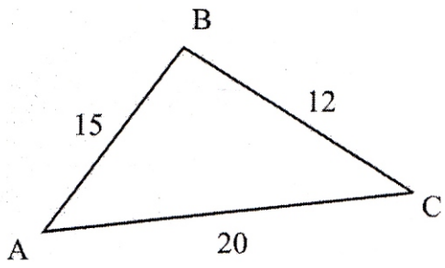
$\therefore \angle B = \cos^{-1}\dfrac{15^2+12^2-20^2}{2(15)(12)} \approx 94.94^\circ$

$15^2 = 12^2 + 20^2 - 2(12)(20)\cos C$

$\therefore \angle C = \cos^{-1}\dfrac{12^2+20^2-15^2}{2(12)(20)} \simeq 48.35^\circ$

$\angle A = 180^\circ - 94.94^\circ - 48.35^\circ$

$\therefore \angle A \simeq 36.71^\circ$



$\angle A = 36.71^\circ$
$\angle B = 94.94^\circ$
$\angle C = 48.35^\circ$

9. Graph the following trigonometric function $y = 2\sin\left(0.5x + \dfrac{\pi}{2}\right) - 3$ on the grid provided below. Show your work. [A 4 marks]

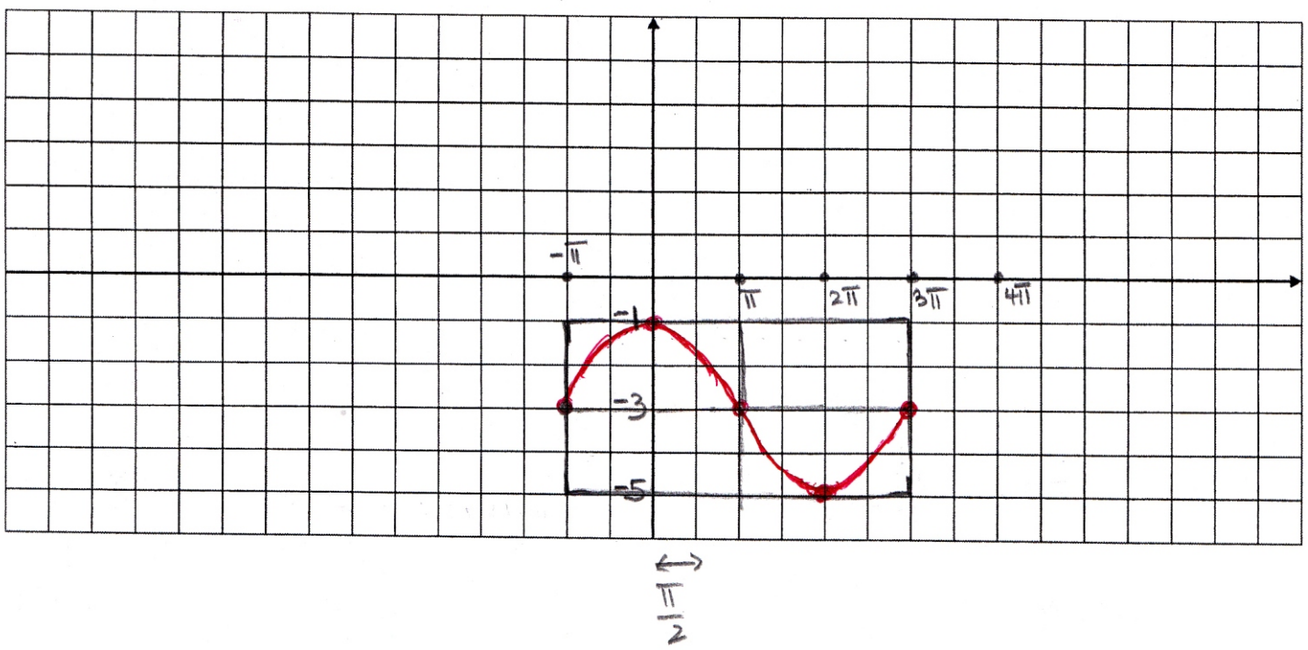
$y_{max} = -1$
$y_{min} = -5$
$y_{axis} = -3$

$0.5x + \frac{\pi}{2} = 0$

$x = -\pi = PS$

$T = \dfrac{2\pi}{0.5} = 4\pi$

$PS + T = -\pi + 4\pi = 3\pi$



$\leftrightarrow \dfrac{\pi}{2}$

10. Determine the equation of the trigonometric function represented graphically on the grid below in the form: $y = a\cos[b(x-c)] + d$. [A 4 marks]

$PS = c = -2\pi$; $T = 4\pi$

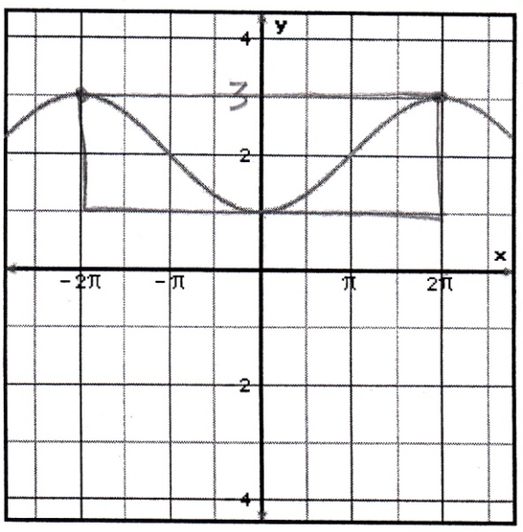
$b = \dfrac{2\pi}{T} = \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

$d = y_{axis} = 2$

$a = A = 1$

$\therefore y = \cos\left[\dfrac{1}{2}(x + 2\pi)\right] + 2$

$= -\cos\dfrac{x}{2} + 2$



11. Describe the transformations. Do not graph. [C 4 marks]

$y = -2\cos(2x - \pi/2) + 3$

$2x - \frac{\pi}{2} = 2\left(x - \frac{\pi}{4}\right)$

- reflection in the x-axis
- vertical expansion by 2
- horizontal stretch by $\frac{1}{2}$
- horizontal translation to the right $\pi/4$
- vertical translation upward by 3

12. Solve for x. [TIPS 2 marks]

$\sin 2x + 2\sin x - 2\cos x = 2$

$2\sin x\cos x + 2\sin x - 2\cos x = 2$

$\sin x(\cos x + 1) = 1 + \cos x$

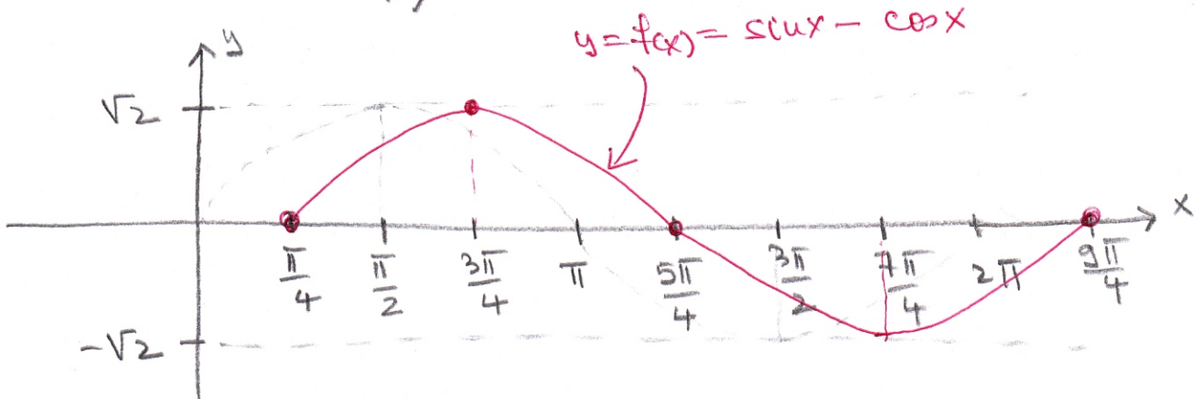
$(\cos x + 1)(\sin x - 1) = 0$

$\cos x = -1$ or $\sin x = 1$

$\therefore\ x = \pi + 2n\pi$ or $x = \frac{\pi}{2} + 2n\pi$

13. Graph $f(x) = \sin x - \cos x$. [TIPS 2 marks]

$f(x) = \frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4}\sin x - \sin\frac{\pi}{4}\cos x\right)$

$= \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$