

These review lessons should not be considered a comprehensive review of all topics. You should be reviewing ALL of your notes, quizzes, tests, and textbook to prepare for the exam/summative.

Unit VI Review - Trigonometric Functions June 14/19

$$y = a \frac{\sin}{\cos} [k(\theta - p.s.)] + v.s.$$

1. Determine the equation of the trigonometric function whose graph has each of the following features.  
**cosine function**

a) An amplitude of 3.5, a period of  $10^\circ$ , an equation of the axis of  $y = 4.5$ , and a horizontal translation of  $66^\circ$ .

$$\text{Per.} = \frac{360^\circ}{k} \quad \therefore k = \frac{360^\circ}{\text{Per.}} = \frac{360^\circ}{10^\circ} = 36$$

$$y = 3.5 \cos [36(\theta - 66)^\circ] + 4.5$$

$$a = 3.5 \quad \text{vs} = +4.5 \quad \text{ps} = +66^\circ$$

**sine function**

b) An amplitude of 8, a period of  $1440^\circ$ , an equation of the axis of  $y = -9$ , and a horizontal translation of  $-270^\circ$ .

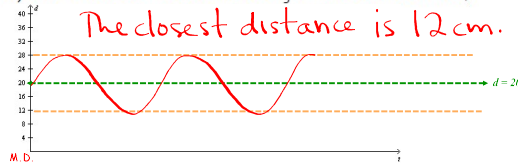
$$\text{Per.} = \frac{360^\circ}{k} = \frac{1}{4} \quad \therefore k = \frac{360^\circ}{1440^\circ} = \frac{1}{4}$$

$$y = 8 \sin \left[ \frac{1}{4}(\theta + 270)^\circ \right] - 9$$

$$a = 8 \quad \text{vs} = -9 \quad \text{ps} = -270^\circ$$

2. A hypnotist is swinging his pocket watch back and forth in front of a motion detector that has just been activated. The distance of the pocket watch from the detector in terms of time is modelled by the function  $d(t) = 8 \sin(180t + 60)^\circ + 20$ , where  $t$  is time in seconds and  $d(t)$  is the distance in cm.

a) What is the closest distance the watch gets to the motion detector?



b) How long does it take for the pocket watch to complete one full cycle of swinging back and forth?

$$\text{Per.} = \frac{360^\circ}{180^\circ} = 2$$

$\therefore$  it takes 2s for one full cycle.

c) What is the distance of the pocket watch from the motion detector at  $t = 10.5$  s?

$$d(10.5) = 8 \sin [180(10.5) + 60]^\circ + 20 = 24$$

$\therefore$  at 10.5s the watch is 24 cm from the detector.

3. Prove.

a)  $\frac{\sin^2 \theta + \cos^2 \theta}{\cot^2 \theta} = \tan^2 \theta$

b)  $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta} = \tan^2 \theta$

$$LS = \frac{1}{\cot^2 \theta}$$

$$= \tan^2 \theta$$

$$= RS$$

$\therefore$  Q.E.D.

$$LS = \frac{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\cos^2 \theta}{\sin^2 \theta}}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\sin^2 \theta + \cos^2 \theta}$$

$$= \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{1}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \tan^2 \theta$$

$$= RS$$

$\therefore$  Q.E.D.

$$* \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$* \sin^2 \theta + \cos^2 \theta = 1$$

$$* \csc \theta = \frac{1}{\sin \theta}$$

$$* \sec \theta = \frac{1}{\cos \theta}$$

$$* \cot \theta = \frac{1}{\tan \theta}$$

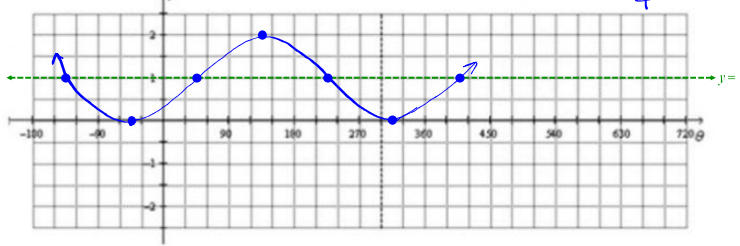
4. Sketch at least one complete cycle of the following.

Important Points every:

a)  $y = -\cos(\theta + 45^\circ) + 1$

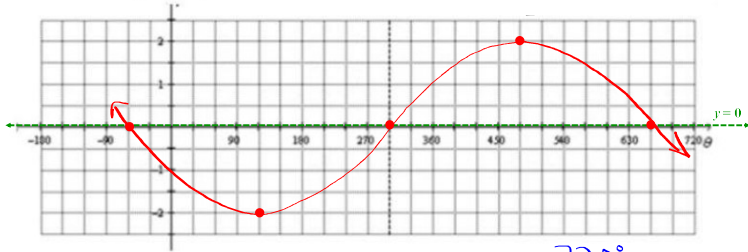
Period:  $360^\circ$

$\frac{360^\circ}{4} = 90^\circ$



b)  $y = -2\sin\left(\frac{1}{2}\theta + 30^\circ\right) = -2\sin\frac{1}{2}(\theta + 60^\circ)$

Period:  $\frac{360^\circ}{\frac{1}{2}} = 720^\circ$



Important Points every:  $\frac{720^\circ}{4} = 180^\circ$

5. The height of a weight on a spring above a table is given by  $h = 20\sin[225^\circ(t - 0.4)] + 50$  where  $h$  is in cm and  $t$  is in seconds. Find the times when the weight is 60 cm above the table during the first 5 seconds. State your answers to two decimal places.



Period:  $\frac{360^\circ}{225^\circ} = 1.6s$

$$60 = 20\sin[225^\circ(t - 0.4)] + 50$$

Let  $a$  represent  $225^\circ(t - 0.4)$ .

$$60 = 20\sin a + 50$$

$$10 = 20\sin a$$

$$\frac{1}{2} = \sin a$$

\*|\*  $RAA = 30^\circ$

$$a = 30^\circ \text{ or } a = 150^\circ$$

replace  $a$  with  $225^\circ(t - 0.4)$

$$225^\circ(t - 0.4) = 30^\circ \quad 225^\circ(t - 0.4) = 150^\circ$$

$$t = 0.5\dot{3}$$

$$t = 1.0\dot{6}$$

➡ Add multiples of 1.6 (the period) to above values to get other times when  $h = 60$  cm.

$\therefore$  it is 60 cm high at 0.53s, 1.06s, 2.13s, 2.6s, 3.73s and 4.26s.