MATHEMATICS GRADE 12 MHF4U, ADVANCED FUNCTIONS **TEST Chapter 5** **EXPONENTIAL AND LOGARITHMIC FUNCTIONS** Teacher: Mr Gugoiu	Date FALL 2008 Name SOLUTIONS	$\frac{\quad}{50}$ =

Questions 1-5 are Multiple-Choice questions [K/U 1 mark each]

1. The expression $10^{-4} = 0.0001$ is equivalent to:

A) $-1 = \log 0.0004$ B) $-4 = \log 0.0001$ C) $10 = \log_4 0.0001$ D) $-10000 = \log 4$ E) $-4 = \log 0.0004$

2. The exact value of $\log_2 \sqrt[3]{2}$ is:

A) 2 B) 3 C) 1/2 D) 1/3 E) 8

3. The solution of the equation $\log x = -2$ is:

A) −20 B) −100 C) 1/100 D) −1/100 E) 100

4. If $\log_{1/a}(1/b) = -1$ then: $\left(\frac{1}{a}\right)^{-1} = \frac{1}{b}$

A) $a = b$ B) $a = 1/b$ C) $ab = -1$ D) $a + b = 0$ E) $a^b = 1$

5. The approximate value of $\log_4 5$ is:

A) −1.61 B) −0.54 C) 1.39 D) 1.61 E) 1.16

Questions 6-10 are True-False questions [K/U 1 mark each]

6. The exponential function is a one-to-one function. T F

7. The inverse function of the logarithmic function is the exponential function.. T F

8. The relation $a^0 = 1$ is true if $a \neq 0$. T F

9. The logarithmic function does not have a vertical asymptote. T F

10. The graph of the function $f(x) = a^x$ does not intersect the x-axis. T F

11. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph. [A 4 marks]

A) [III] $f(x) = -2^x$

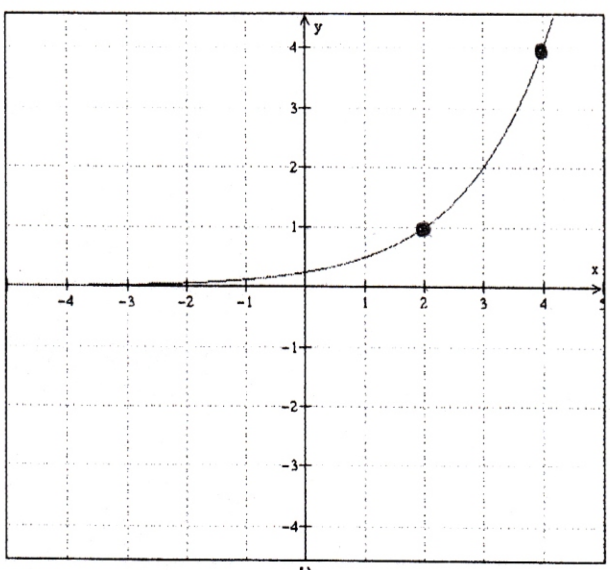
B) [ ] $g(x) = 2^{(x+2)}$

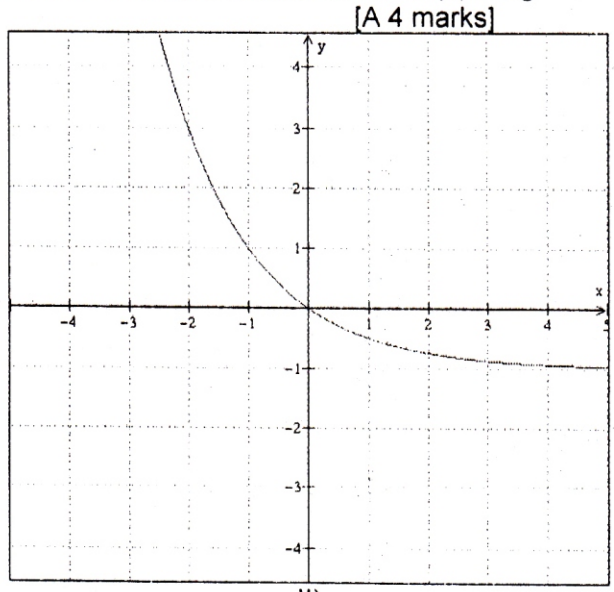
C) [IV] $h(x) = 3^x$

D) [I] $k(x) = 2^{(x-2)}$

E) [ ] $p(x) = 0.5^x + 2$

F) [II] $q(x) = 0.5^x - 1$


I)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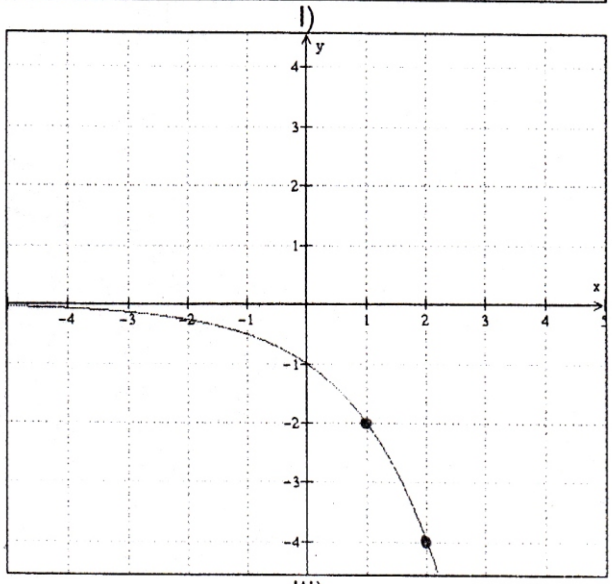
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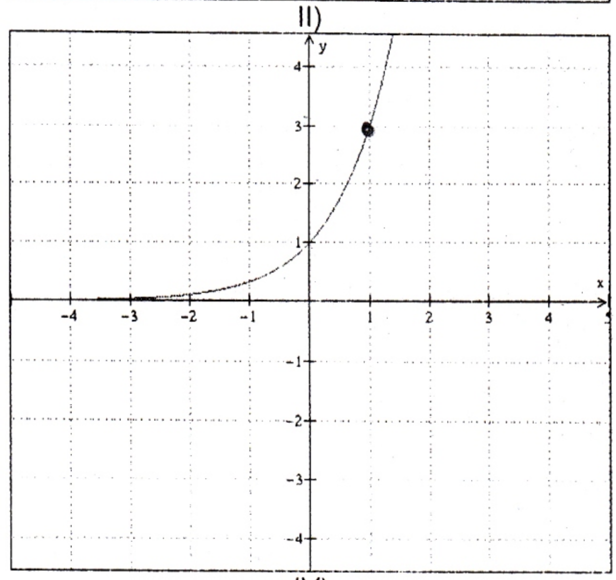

IV)

The following questions are long answer questions. Show your work to get full marks.

12. [K/U 2 marks]

a) Convert the exponential notation into the logarithmic notation.

$\sqrt{2}^{8} = 16$

$\therefore\ 8 = \log_{\sqrt{2}} 16$

a) Convert the logarithmic notation into the exponential notation.

$\log_3 81 = 4$

$\therefore\ 3^4 = 81$

13. Solve for x. [K/U 6 marks]

[1] a) $2^x = 65536$

$x \ln 2 = \ln 65536$

$x = \dfrac{\ln 65536}{\ln 2}$

$\therefore\ x = 16$

[1] b) $\log x = -5$

$10^{-5} = x$

$x = 10^{-5} = 0.00001$

Restrictions : $x > 0$ (true) [0.25]

$\therefore\ x = 10^{-5} = 0.00001$

[2] c) $5^{x^2-2} = 25$

$5^{x^2-2} = 5^2$

$x^2 - 2 = 2$

$x^2 = 4$

$\therefore\ x = \pm 2$

[2] d) $\ln(x^2 + x) = 0$

$\ln(x^2 + x) = \ln 1$

$x^2 + x = 1$

$x^2 + x - 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1+4}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$ ✓

$\therefore\ x = \dfrac{-1 \pm \sqrt{5}}{2}$

Restrictions: $x^2 + x > 0$

$x < -1$ or $x > 0$ (true) [0.25]

-1 0

14. Find the domain of the following functions: [K/U 4 marks]

[2] a) $f(x) = \dfrac{1}{10^x - 1}$

$10^x - 1 \neq 0$

$10^x \neq 1$

$\therefore\ x \neq 0$

[2] a) $g(x) = \log(1 - \sqrt{x})$

$1 - \sqrt{x} > 0$

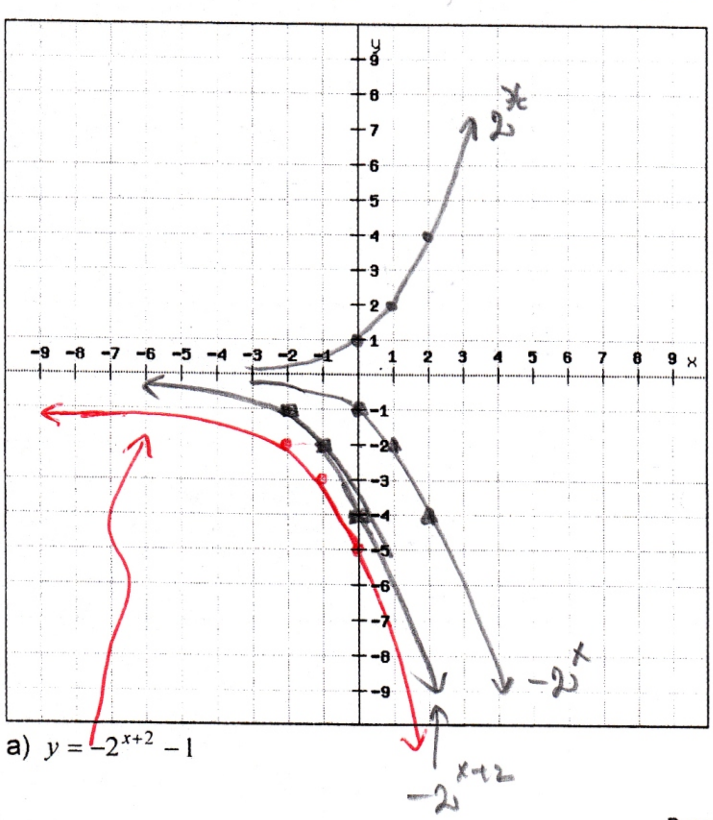
$1 > \sqrt{x}$

$\sqrt{x} < 1 \iff x < 1$

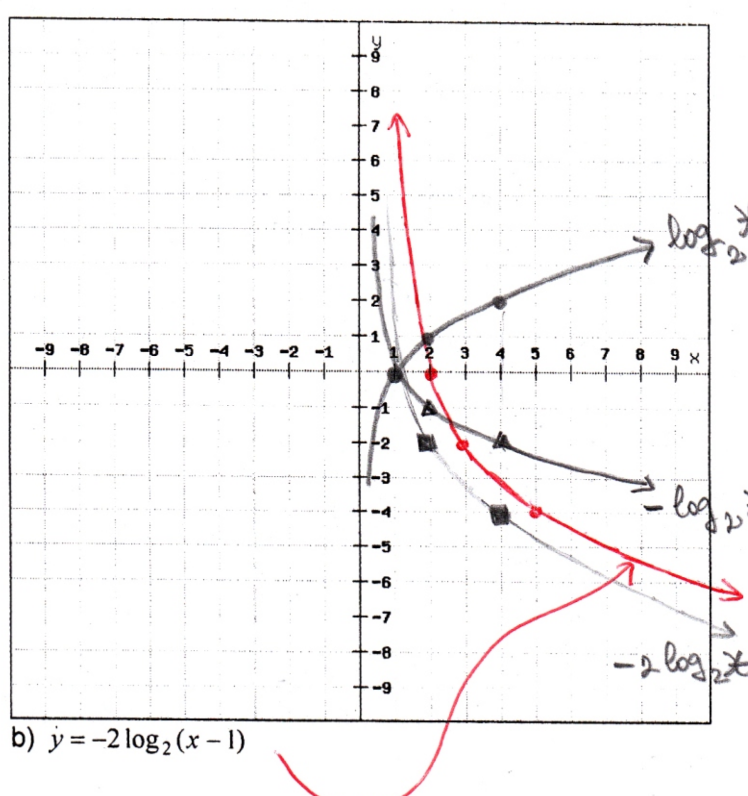
$x \geq 0$

$\therefore\ x \in [0, 1)$

15. Use transformations to graph the following functions. [K/U 4 marks]



a) $y = -2^{x+2} - 1$



b) $y = -2\log_2(x-1)$

16. Find the exact value of: $\log_a \frac{1}{\sqrt{a}} + \log_a \sqrt[3]{a} - \frac{1}{3}\log_a a^2$ [A 3 marks]

$\log_a a^{-1/2} + \log_a a^{1/3} - \frac{1}{3}\log_a a^2$

$= -\frac{1}{2} + \frac{1}{3} - \frac{2}{3} = -\frac{1}{2} - \frac{1}{3} = -\frac{5}{6}$

17. Solve for x: [A 6 marks]

[3] a) $64^x - 10(8^x) + 16 = 0$

$8^x = y$

$y^2 - 10y + 16 = 0$

$y = \frac{10 \pm \sqrt{100-64}}{2}$

$= \frac{10 \pm 6}{2} \begin{smallmatrix} \nearrow 8 \\ \searrow 2 \end{smallmatrix}$

$8^x = 8 \Rightarrow x = 1$

$8^x = 2 \Leftrightarrow 2^{3x} = 2^1$

$\Leftrightarrow 3x = 1 \Leftrightarrow x = \frac{1}{3}$

$\therefore x = 1$ or $x = \frac{1}{3}$

[3] a) $-\ln 3 + \ln(2x-1) = -\ln 4 + \ln(x+1)$

$\ln \frac{2x-1}{3} = \ln \frac{x+1}{4}$

$\frac{2x-1}{3} = \frac{x+1}{4}$

$4(2x-1) = 3(x+1)$

$8x - 4 = 3x + 3$

$5x = 7$

$x = \frac{7}{5} = 1.4$

Restrictions

$2x - 1 > 0 \Rightarrow x > \frac{1}{2} = 0.5$ (yes)

$x + 1 > 0 \Rightarrow x > -1$ (yes)

$\therefore x = \frac{7}{5} = 1.4$

0.5 0.5 1.5 0.5

18. Find the domain, range, x- and y-intercept for the following functions. Do not graph. [A 6 marks]

[3] a) $f(x) = (-3)2^{x-1} + 4$

$\therefore D = \mathbb{R}$

$\therefore R = (-\infty, 4)$

$\therefore$ y-int $= (-3)\, 2^{-1} + 4 = -\frac{3}{2} + 4 = \frac{5}{2}$

x-int

$0 = (-3)\, 2^{x-1} + 4$

$(3)\, 2^{x-1} = 4$

$2^{x-1} = \frac{4}{3}$

$(x-1)\ln 2 = \ln(4/3)$

$\therefore x = 1 + \frac{\ln(4/3)}{\ln 2}$

[3] a) $g(x) = (-0.5)\log_2(x+1) + 3$

$D = ?$

$x + 1 > 0 \Rightarrow x > -1$

$\therefore D = (-1, \infty)$

$\therefore R = \mathbb{R}$

y-int $= g(0) = -0.5 \times \log_2 1 + 3$

$\therefore$ y-int $= 3$

x-int

$0 = (-0.5)\log_2(x+1) + 3$

$0.5 \log_2(x+1) = 3$

$\log_2(x+1) = 6$

$2^6 = x+1$

$\therefore x = 63$

19. A model for the number of bacteria in a culture after t hours is given by $P(t) = P_0 e^{kt}$. After 3 hours it is observed that 300 bacteria are present. After 10 hours it is observed that 1000 bacteria are present. [A 5 marks]

[1] a) What is the initial number of bacteria?

$$300 = P_0 e^{k3h}$$
$$1000 = P_0 e^{k10h}$$
$$\frac{10}{3} = e^{7kh}$$
$$e^{kh} = \left(\frac{10}{3}\right)^{1/7}$$
$$kh = \frac{1}{7} \ln(10/3)$$

$$P_0 = \frac{300}{e^{3kh}} = \frac{300}{\left(\frac{10}{3}\right)^{3/7}} = 179$$

$\therefore P_0 = 179$
The initial number of bacteria is 179

[1] b) What is the doubling period (the time after which the number of bacteria is doubled) of these bacteria?

$$2\cancel{P_0} = \cancel{P_0} e^{kD}$$
$$\ln 2 = kD$$

$$2\cancel{P_0} = \cancel{P_0} e^{kt} \rightarrow t = \frac{\ln 2}{k} = \ldots$$
$$\ln 2 = kt \rightarrow$$

$$D = \frac{\ln 2}{k} = \frac{\ln 2}{\frac{1}{7}\ln(10/3)} h = \frac{7 \ln 2}{\ln(10/3)} h = 4.03 \text{ hours}$$

$\therefore D = 4.03$ hours. The doubling period is 4.03 hours

[1] c) How many bacteria will be after 15 hours?

$t = 15h$
$$P = P_0 \times e^{k15h} = 179 \times \left(\frac{10}{3}\right)^{15/7} = 2362$$

$\therefore P = 2362$
After 15 hours there will be 2362 bacteria

[1] d) What is the time after which the number of bacteria is 2000?

$$2000 = 179 \times e^{kt}$$
$$\ln \frac{2000}{179} = kt$$
$$t = \frac{\ln(2000/179)}{\frac{1}{7}\ln(10/3)} h = \frac{7 \times \ln(2000/179)}{\ln(10/3)} h = 14.03 \text{ hours}$$

$\therefore t = 14.03$ hours. After 14.03 hours, there will be 2000 bacteria.

[1] e) What is the growth rate (percentage per hour) of these bacteria?

$$r = \frac{P(1h) - P_0}{P_0} = \frac{P_0 e^{kh} - P_0}{P_0} =$$
$$= e^{kh} - 1 = \left(\frac{10}{3}\right)^{1/7} - 1 = 18.77\%$$

$\therefore r = 18.77\%$
The growth rate is 18.77% per hour.